An asymptotic-safety mechanism for chiral Yukawa systems

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We introduce Weinberg's idea of asymptotic safety and pave the way towards an asymptotically safe chiral Yukawa system with a $U(N_{\mathrm{L}})_{\mathrm{L}} \otimes U(1)_{\mathrm{R}}$ symmetry in a leading-order derivative expansion using nonperturbative functional RG equations. As a toy model sharing important features with the standard model we explicitely discuss $N_{\mathrm{L}} = 10$ for which we find a non-Gaußian fixed point and compute its critical exponents. We observe a reduced hierarchy problem as well as predictions for the toy Higgs and the toy top mass.

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1. Introduction

Quantum field theory (QFT) has been very successful in the description of a large number of phenomena, in particular in high energy physics. However there is a widespread belief that QFT in many cases only accounts for effective theories and that it is not suited to constitute a fundamental theory but should be replaced by another concept at a microscopic scale. This is due to the apparent non-renormalisability of important action functionals, e.g. the Einstein-Hilbert action, describing gravity [1, 2, 3, 4]. Furthermore the standard model of particle physics is plagued by the problem of triviality in the sector describing quantum electrodynamics (QED) [5, 6, 7, 8] and in the Higgs sector [9, 10, 11, 12, 13, 14], which forbids to extend the standard model beyond a certain ultraviolet scale. However the issue of non-renormalisability is often adressed within *perturbative* QFT, which is not an

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essential concept of QFT itself. For perturbative QFT to apply it has to be possible to expand about vanishing interactions, i.e., about the Gaußian fixed-point (GFP), which is a severe limitation. A more general conceptual understanding of QFT, not sticking to the constraint of perturbativity, was elucidated by Steven Weinberg when he introduced the idea of asymptotic safety [15, 16, 17]. In an asymptotically safe QFT the microscopic action entering the functional integral approaches an interacting fixed point in the space of action functionals in the infinite UV cutoff limit. Thus no unwanted divergencies can occur. This renders the theory well-defined on all scales. The scenario has already been applied to a number of models ranging from four-fermion models [18, 19, 20], simple Yukawa systems [21, 22], nonlinear sigma models in $d > 2$ [23], extra-dimensional gauge theories [24], and gravity [25, 26, 27, 28, 29].

The idea of asymtpotic safety has gained considerable attention in the context of gravity, especially in the last 10 years. In fact, there is a lot of evidence that such an interacting fixed point exists for diffeomorphism invariant actions, allowing for a formulation of a non-perturbative renormalisable quantum field theory of gravity [25, 26, 27, 28]. The calculations necessary to compute the RG flow in theory space, however, are tedious and involve a number of techniques, such as e.g. the background field method, that complicate the discussion of the bare concept of asymptotic safety considerably. It would therefore be convenient to have a simpler setting, where the idea and the basic concepts of asymptotic safety could be tested and understood in a transparent manner.

On the quest for such a simpler setting the standard model with its problem of triviality might be a beacon. Here, the Landau poles of perturbation theory in the QED and the Higgs sector suggest that one should introduce new degrees of freedom for a fundamental description of particle physics. However, before doing so a non-perturbative computation of the Higgs sector including fermions and also gauge fields would have to show whether the problem of triviality still persists or whether the theory might be asymptotically safe in Weinberg's sence, by acquiring a fixed point in the ultraviolet. As a step towards this scenario, we investigate a toy model for the standard model without gauge fields and with a particular chiral left/right asymmetry [22]. To leading order in the derivative expansion, we find an asymptotically safe theory which is well-defined on all scales (renormalisable) with a highly predictive power and comparatively simple from a computational point of view. Apart from a possible application to the complete standard model of particle physics including a prediction of the Higgs and the top mass, it also allows for a better understanding of how asymptotic safety works.

This contribution is organised as follows: in section 2 we briefly recall

triviality and the hierarchy problem as they occur in the Higgs sector of the standard model. Section 3 introduces the idea of asymptotic safety and the Wetterich equation, which is our tool to investigate QFT non-perturbatively. In section 4 we discuss our toy model, a particular chiral Yukawa system, whose fixed-point structure is analysed in section 5. Conclusions are drawn in the last section.

2. Two problems of the Higgs sector

In the Higgs sector of the standard model we find the problem of triviality and the hierarchy problem. To understand the nature of the triviality problem, it suffices to consider a purely bosonic theory, where we mimic the Higgs field in terms of a single component real scalar field ϕ, with a Lagrangian

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 + \frac{m^2}{2}\phi^2 + \frac{\lambda}{8}\phi^4. \tag{1}$$

Using one-loop RG-improved perturbation theory we can establish a relation between the bare and the renormalized four-Higgs-boson coupling $\lambda\phi^4$

$$\frac{1}{\lambda_{\mathrm{R}}} - \frac{1}{\lambda_\Lambda} = \beta_0 \ \mathrm{Log}\left(\frac{\Lambda}{m_{\mathrm{R}}}\right), \ \beta_0 = \mathrm{const.} > 0, \tag{2}$$

where λ_Λ and λ_{R} are the bare and the renormalised couplings, respectively, Λ is the ultraviolet cutoff scale and m_{R} the renormalised mass. For fixed non-zero (non-trivial) λ_{R} and m_{R} the bare coupling λ_Λ runs into a pole at a finite UV scale, the Landau pole Λ_{L}. This indicates the breakdown of the perturbative QFT at this scale.

For a purely bosonic theory also a nonperturbative treatment of QFT (e.g., on the lattice) confirms triviality, inhibiting the theory to be fundamental. However the standard model Higgs sector of course contains bosonic and fermionic degrees of freedom; as we show in Sect. 5, a balancing of the fermionic and bosonic quantum fluctuations can make the pole disappear and render the system well-defined on all scales.

Whereas the triviality problem is a conceptual problem, the hierarchy problem is only a problem of the unnaturalness of strongly fine-tuned initial conditions. We observe a huge hierarchy in the standard model between the electroweak scale $\Lambda_{\mathrm{EW}} \sim 10^2$GeV and e.g. the scale of a grand unified theory $\Lambda_{\mathrm{GUT}} \sim 10^{16}$GeV. The Higgs mass renormalises quadratically with the UV cutoff Λ and in perturbation theory the relation between bare and renormalised mass is given by

$$m_{\mathrm{R}}^2 \sim m_{\Lambda,\mathrm{UV}}^2 - \delta m^2. \tag{3}$$

Here, the renormalised mass is of order $m_{\mathrm{R}}^2 \sim 10^4 \mathrm{GeV}^2$, and the fluctuation contribution is of the order of the cutoff $\Lambda^2 \sim \delta m^2 = X \cdot 10^{32} \mathrm{GeV}^2$ (here, X is a pure number depending on the coupling values). As a consequence, the counterterm has to be $m_{\Lambda,\mathrm{UV}}^2 = 10^{32}(X + ...10^{-28})\mathrm{GeV}^2$. It is this second term in parentheses which has to be fine-tuned to a relative precision of $\Lambda_{\mathrm{EW}}^2/\Lambda_{\mathrm{GUT}}^2 \sim 10^{-28}$. In an RG language this quadratical cutoff dependence corresponds to a renormalisation at a noninteracting (Gaußian) fixed point with a critical exponent $\Theta = 2$. A nonperturbative analysis with an interacting fixed point might have small critical exponents, e.g. close to zero, which could lead to a weaker cutoff dependence and the chance to make the hierarchy problem disappear. We present such a computation with a reduced hierarchy problem in Sect. 5.

3. Asymptotic safety and the flow equation

In this section we sketch the idea of asymptotic safety, comprehensive reviews on asymptotic safety can be found in [16, 29]. Consider an effective average action $\Gamma_k[\chi]$ of an effective quantum field theory at scale k. This is an action functional in the set of fields χ and consists of operators which are compatible with the underlying symmetries. $\Gamma_k[\chi]$ contains all the fluctuations of the quantum fields with momenta larger than k. It can be understood as an effective theory where a tree level evaluation suffices to describe physics at scale k. We can think of $\Gamma_k[\chi]$ as an expansion in terms of (dimensionless) running couplings $g_{i,k}$ and all possible field operators $\mathcal{O}_i$.

$$\Gamma_k[\chi] = \sum_i g_{i,k}\mathcal{O}_i, \text{ e.g. } \mathcal{O}_i = \left\{\chi^2, \chi^4, (\partial\chi)^2, \dots\right\} . \tag{4}$$

The dependence of the effective action on the scale k, i.e. the renormalization group flow, is given by the β functions of the running couplings,

$$\partial_t \Gamma_k[\chi] = \sum_i \beta_{i,k}\mathcal{O}_i \, , \text{where } \beta_{i,k} = \partial_t g_i \text{ and } \partial_t = k\frac{d}{dk} \, . \tag{5}$$

The field operators span the theory space, as is shown in Fig. 1. On the left hand side of Fig. 1 a sketch of the RG flow is given. The position of the effective average action $\Gamma_k[\chi]$ in theory space is given by a set of coordinates of running couplings $\{g_{i,k}\}$. As we lower the scale from k to $k - \Delta k$ by an RG step, the transformation of the running couplings and so the change of position in theory space is described by the β functions and we end up at a different effective average action $\Gamma_{k-\Delta k}[\chi]$ at the new scale.

Suppose there is a (possibly non-Gaußian) fixed point in theory space (see r.h.s. of figure 1) where $\beta_{i,k} = 0 \ \forall\ i$. If we can find an RG trajectory

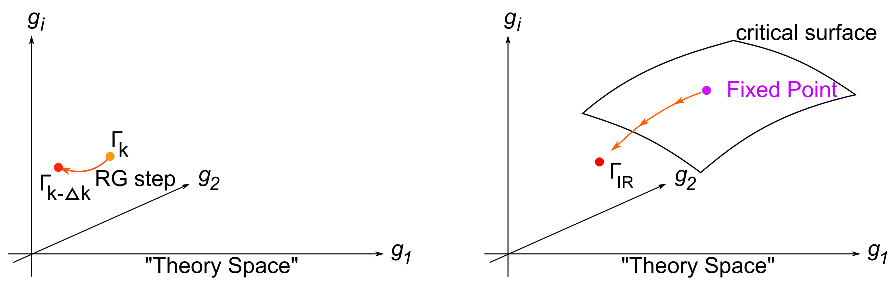

Fig. 1. Sketch of a 3-dimensional subspace of theory space spanned by three operators with associated couplings g_1, g_2 and g_i. Left panel: an RG step shifts the effective average action at a scale k to a different point at a lower scale. Right panel: RG flow from an ultraviolet fixed point to a physical infrared effective action.

which connects the fixed point with a meaningful physical theory represented by an effective average action $\Gamma_{\rm IR}$ at some infrared scale, then we have found a quantum field theory, which can be extended to arbitrarily high scales, since for the cutoff scale $\Lambda \to \infty$ we just run into the fixed point and no pathological divergencies can appear. This solves the triviality problem. Note that in the perturbative setting of the standard model, the only fixed point is the Gaußian fixed point, which is not connected to a physically sensible (non-trivial) effective action in the IR.

In the vicinity of a fixed point $g^* = \{g_i^*\}$ we can study the behaviour of the RG trajectories near the fixed point, using the linearized flow equations

$$\partial_t g_i = B_i{}^j (g_j - g_j^*),\ \ B_i{}^j = \frac{\partial \beta_i}{\partial g_j}\Big|_{g^*} + \mathcal{O}((g-g^*)^2). \tag{6}$$

The solution reads

$$g_i = g_i^* + \sum_I C_I V_i^I \left(\frac{k_0}{k}\right)^{\Theta_I}, \tag{7}$$

where the integration parameters C^I define the initial conditions at a reference scale k_0. Furthermore, the eigenvectors V^I and the negative of the eigenvalues Θ^I of the stability matrix B_i^j satisfy $B_i{}^j V_j^I = -\Theta^I V_i^I$. For the flow towards the UV, the directions in theory space with $\mathrm{Re}\{\Theta_I\} > 0$ (relevant directions) are attracted towards the fixed point, see Fig. 2, left panel. The set of trajectories which run into the fixed point is called the critical surface S. The number of linearly independent relevant directions at the fixed point corresponds to the dimension of S. These directions determine the physics in the infrared and the number of physical parameters to

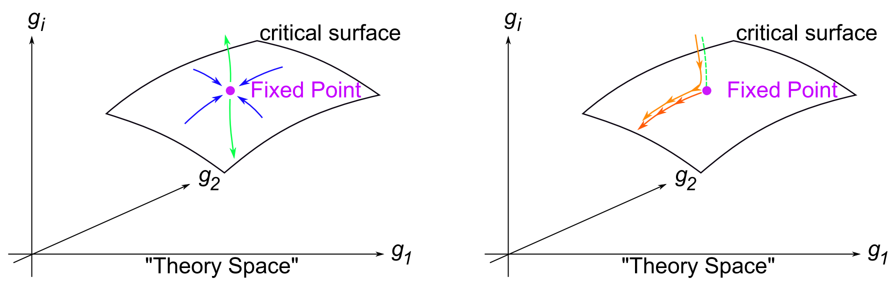

Fig. 2. Left panel: flow towards the UV in the vicinity of the fixed point. Relevant directions approach the fixed point (blue arrows) and span the critical surface; irrelevant directions run away from the fixed point for $k \to \infty$ (green arrows). Right panel: flow towards the IR. As the irrelevant directions are strongly attracted to the critical surface, the IR observables of the theory are solely determined by the relevant directions.

be fixed. The theory is predictive if dim(S) is finite. The directions with $\mathrm{Re}\{\Theta_I\} < 0$ run away from the fixed point as we increase k, see Fig. 2, left panel. By contrast, as we decrease k, the irrelevant directions rapidly approach the critical surface, as displayed in Fig. 2, right panel. Therefore, the observables in the IR are all dominated by the properties of the fixed point, independently of whether the flow has started exactly in or near the critical surface. This establishes the predictive power of the asymptotic safety scenario.

If a critical exponent is much larger than zero, say of $\mathcal{O}(1)$, the RG trajectory rapidly leaves the fixed-point regime towards the IR. Therefore, separating a typical UV scale where the system is close to the fixed point from the IR scales where, e.g., physical masses are generated requires a significant fine-tuning of the initial conditions. In the context of the standard model, the size of the largest Θ^I is a quantitative measure of the hierarchy problem.

As the non-perturbative tool to search for a NGFP in the space of action functionals and to compute the properties at this FP, we use the Wetterich equation which provides a vector field β in theory space in terms of RG β functions. The flow of the effective average action Γ_k is determined by [30]:

$$\partial_t \Gamma_k[\chi] = \frac{1}{2}\mathrm{STr}\{[\Gamma_k^{(2)}[\chi] + R_k]^{-1}(\partial_t R_k)\}. \tag{8}$$

Here, $\Gamma_k^{(2)}$ is the second functional derivative with respect to the field χ. The function R_k denotes a momentum-dependent regulator that suppresses

IR modes below a momentum scale k. The solution to the Wetterich equation provides for an RG trajectory in theory space, interpolating between the bare action S_Λ to be quantized $\Gamma_{k\to\Lambda} \to S_\Lambda$ and the full quantum effective action $\Gamma = \Gamma_{k\to 0}$, being the generating functional of 1PI correlation functions; for reviews, see [31].

4. A chiral Yukawa system

As a toy model for the standard model we employ a Yukawa theory with chiral fermions including one right-handed fermion ψ_{R} and N_{L} left-handed fermions ψ_{L}^a, which are coupled to N_{L} complex bosons ϕ^a via a simple Yukawa interaction term $\bar{h}_k$ [22]. The theory space is truncated by an action functional in *leading-order* derivative expansion and reads

$$\begin{aligned}\Gamma_k \;=\; \int d^dx \Big\{ & i(\bar{\psi}_L^a \partial\!\!\!/ \psi_L^a + \bar{\psi}_R \partial\!\!\!/ \psi_R) + (\partial_\mu \phi^{a\dagger})(\partial^\mu \phi^a) \\ & + U_k(\rho) + \bar{h}_k \bar{\psi}_R \phi^{a\dagger} \psi_L^a - \bar{h}_k \bar{\psi}_L^a \phi^a \psi_R \Big\}, \end{aligned} \tag{9}$$

where we define $\rho = \phi^{a\dagger}\phi^a$. The index k at the Yukawa coupling and the effective potential shall indicate their scale dependence, which will be governed by the Wetterich equation. The action is invariant under chiral $U(N_{\mathrm{L}})_{\mathrm{L}} \otimes U(1)_{\mathrm{R}}$ transformations. Additionally to the *perturbative* complications of triviality and the hierarchy problem, this toy model also shares the feature of a left-handed chiral sector with the Higgs-top sector of the standard model. The number of left-handed fermions N_{L} is left as a free parameter in order to study the dependence of a potential fixed point on a varying number of degrees of freedom. Various models with Yukawa-type interactions have already been studied within this derivative expansion technique and yielded reliable results in low-energy QCD [32], critical phenomena [33], and ultra-cold fermionic atom gases [34].

For the analysis of the fixed-point structure we introduce dimensionless quantities

$$\tilde{\rho} \;=\; k^{2-d}\rho, \quad h^2 = k^{d-4}\bar{h}_k^2, \quad u(\tilde{\rho}) = k^{-d}U_k(\rho)|_{\rho=k^{d-2}\tilde{\rho}}. \tag{10}$$

The dimensionless effective potential u is expanded about its dimensionless minimum $\kappa := \tilde{\rho}_{\mathrm{min}} > 0$,

$$u \;=\; \frac{\lambda_2}{2!}(\tilde{\rho}-\kappa)^2 + \frac{\lambda_3}{3!}(\tilde{\rho}-\kappa)^3 + \ldots \text{ with } \kappa,\ \lambda_{n_{\max}},\ \lambda_2 > 0. \tag{11}$$

This potential describes a theory in the regime of spontaneously broken symmetry (SSB), which is depicted in figure 3. The constraints formulated

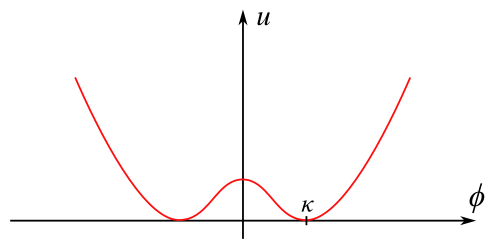

Fig. 3. Sketch of the scale-dependent effective potential u as a function of the field ϕ_a. For a theory in the regime of spontaneously broken symmetry we employ an expansion about a positive minimum at κ, corresponding to a nonvanishing vacuum expectation value (vev) for the bosonic field.

in the expansion of the effective potential make sure that it is bounded from below and constitutes an expansion about a positive minimum. We could also think of an expansion about vanishing minimum, which would describe the theory in the symmetric regime. However, the existence of a suitable FP in the symmetric regime has been ruled out within the validity limits of the derivative expansion [22].

5. Fixed points and critical exponents

To understand the occurence of fixed points in this model we investigate the loop contributions to the running of the dimensionless version of the squared bosonic field expectation value κ in its flow equation of the form

$$\partial_t \kappa = -2\kappa + \text{bosonic interactions} - \text{fermionic interactions}. \tag{12}$$

A positive sum of the contribution from the interaction terms gives rise to a fixed point at $\kappa > 0$ and allows for asymptotic safety, as we demonstrate below. For a negative sum, no fixed point is possible [22]. Since fermions and bosons contribute with opposite signs to the interaction terms, the existence of a fixed point $\kappa^* > 0$ crucially depends on the relative strength between bosonic and fermionic fluctuations, as is sketched in Fig. 4. In this figure the solid line depicts the free massless theory with a trivial Gaußian fixed point at $\kappa = 0$. If the fermions dominate, the interaction terms are negative and the fixed point is shifted to negative values (being irrelevant for physics), cf. dotted line. If the bosonic fluctuations dominate, the κ flow develops a non-Gaußian fixed point at positive values $\kappa^* > 0$, cf. dashed line. This fixed point is UV attractive, implying that the vev is a relevant operator near the fixed point. If the interaction terms are approximately κ independent, the slope of $\partial_t \kappa$ near the fixed point is still close to -2, corresponding to a critical

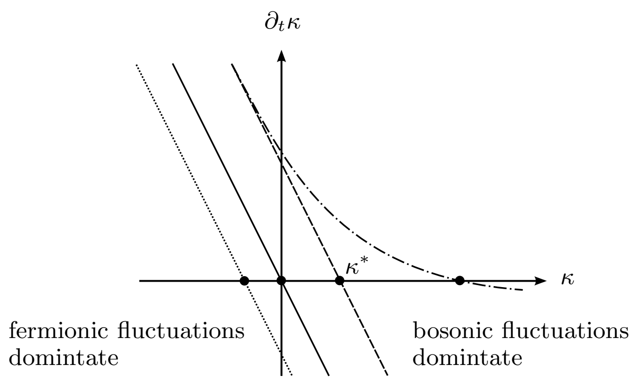

Fig. 4. Sketch of the β-function for the dimensionless squared Higgs vacuum expectation value κ. Dominating fluctuations of the boson field allow for a positive κ^* (dashed line) and a suitable κ-dependence flattens the β-function near the fixed-point, which reduces the hierachy problem (dot-dashed line).

exponent $\Theta \simeq 2$ and a persistent hierarchy problem. An improvement of "naturalness" could arise from a suitable κ dependence of the interaction terms that results in a flattening of the κ flow near the fixed point, cf. dot-dashed line.

The fixed point in the SSB regime induces a new mechanism for asymptotic safety: near the fixed point, the vev exhibits a conformal behavior, being always proportional to the actual RG scale k. If the vev was proportional to a fixed threshold scale (as is usually the case during a symmetry-breaking transition), the vev would induce the decoupling of massive modes below this threshold. But as the vev runs with the scale, the system is always in the onset domain of these threshold effects without showing any decoupling. In other words, this conformal-vev mechanism renders the threshold effects strong enough to induce the fixed point, but at the same time weak enough in order to avoid decoupling.

For the conformal-vev mechanism to appear, the bosonic and fermionic contributions have to be balanced. Whether or not this is possible depends on the degrees of freedom and the algebraic structure of the theory: in Fig. 5 we show the loop contributions from the bosonic and fermionic fluctuations, which are particular for our model. The left loop involves only inner boson lines. The vertex λ_2 allows for a coupling between all available boson components. This implies a linear dependence on N_{L} for the renormalization of the boson contribution. For the fermion loop on the r.h.s of figure 5 the incoming boson ϕ_a fully determines the structure and does not allow for other left-handed inner fermions than ψ^a_{L}, inhibiting an N_{L} dependence of

this loop.

For a systematic analysis of the fixed-point structure let us start with a very basic truncation only involving the flowing minimum of the effective potential κ, the four-boson interaction λ_2 and the Yukawa coupling h. For the fixed points we have to solve a set of nonlinear algebraic equations of the form

$$\partial_t h^2 = \beta_h = 0, \tag{13}$$
$$\partial_t \lambda_2 = \beta_\lambda = 0, \tag{14}$$
$$\partial_t \kappa = \beta_\kappa = 0. \tag{15}$$

The β functions for the couplings κ and λ_2 can be computed from the effective potential flow, yielding

$$\beta_\kappa = -2\kappa + \frac{(2N_{\mathrm{L}}-1)}{32\pi^2} + \frac{3}{32\pi^2(1+2\kappa\lambda_2)^2} - \frac{h^2}{4\pi^2\lambda_2(1+\kappa h^2)^2}, \tag{16}$$
$$\beta_\lambda = \frac{(2N_{\mathrm{L}}-1)\lambda_2^2}{16\pi^2} + \frac{9\lambda_2^2}{16\pi^2(1+2\kappa\lambda_2)^3} - \frac{h^4}{2\pi^2(1+\kappa h^2)^3}, \tag{17}$$

and the β function of the Yukawa coupling reads

$$\begin{aligned}\beta_h = \frac{1}{16\pi^2}\frac{h^4}{(1+\kappa h^2)}\Bigg\{ &-\frac{6\kappa\lambda_2}{(1+2\kappa\lambda_2)^2}\left(\frac{1}{1+\kappa h^2}+\frac{2}{1+2\kappa\lambda_2}\right) \\ &-\left(\frac{1}{1+\kappa h^2}+1\right)+\frac{1}{(1+2\kappa\lambda_2)}\left(\frac{1}{1+\kappa h^2}+\frac{1}{1+2\kappa\lambda_2}\right) \\ &+\frac{2\kappa h^2}{(1+\kappa h^2)}\left(\frac{2}{1+\kappa h^2}+1\right)+2\lambda_2\kappa\left(\frac{1}{1+\kappa h^2}+2\right) \\ &-\frac{2\kappa h^2}{(1+\kappa h^2)(1+2\kappa\lambda_2)}\left(\frac{2}{1+\kappa h^2}+\frac{1}{1+2\kappa\lambda_2}\right)\Bigg\}.\end{aligned} \tag{18}$$

The explicit derivation of these β functions using the Wetterich equation together with an optimised regulator[35] can be found in [22]. With these β functions we find non-Gaußian fixed points (NGFPs) for $1 \leq N_{\mathrm{L}} \leq 57$. An extension of the truncation in the effective potential shows a reliable convergence of the fixed point and its critical exponents [22]. An extension of the fixed-point analysis with a truncation to next-to-leading order in the derivative expansion by introducing flowing wave function renormalisations for the left-/right-handed fermion fields as well as for the boson field is not straightforward, since the algebraic structure of the β functions becomes more involved. We have found no analytical way to solve those equations.

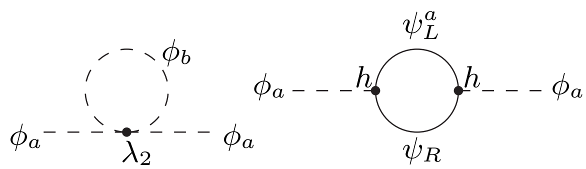

Fig. 5. Loop contributions to the renormalization flow of the Higgs dimensionless vev. The loop on the l.h.s. couples all available boson components giving a linear dependence on N_L. This is not the case for the fermionic loop on the r.h.s. which is fully determined by the incoming boson field component.

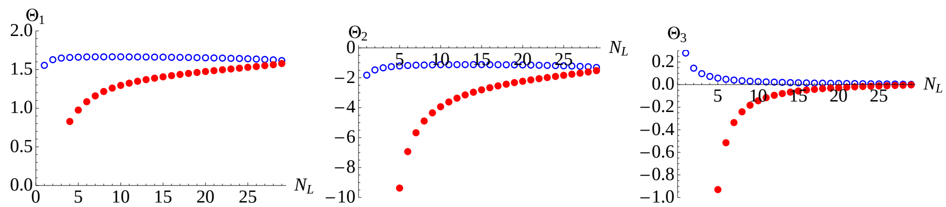

Fig. 6. Critical exponents for the NGFPs in the leading order truncation as a function of N_L. The fixed point corresponding to the open circles has two relevant directions, whereas the fixed point corresponding to the filled circles has only one relevant direction.

Numerical evidence suggests that the fixed point might be destabilized by large anomalous dimensions at next-to-leading order. This is not surprising, as many massless Goldstone and fermion modes exist in the present model also in the SSB regime which can induce instabilities. As the standard model does not have these massless modes, it is natural to expect that this issue at next-to-leading order is resolved by introducing gauge fields. In Fig. 6 we show the critical exponents of the basic three parameter truncations as a function of the left-handed fermion number N_L. As an explicit example to show how asymptotic safety leads to predictivity for toy standard model observables we study a leading-order truncation expanded up to $\frac{\lambda_6}{6!}\rho^6$ in the effective potential and $N_\mathrm{L} = 10$. We find (non-universal) fixed-point values

$$\kappa^* = 0.0152, \quad \lambda^* = 12.13, \quad h^{*2} = 57.41.$$

For the universal critical exponents we obtain

$$\Theta_1 = 1.056, \quad \Theta_2 = -0.175, \quad \Theta_3 = -2.350.$$

There is only one relevant direction, corresponding to one physical parameter to be fixed. All other parameters are predictions of the theory. The exponent of the relevant direction is 1.056 (as compared to 2 near the Gaußian

fixed point), such that the hierarchy problem is weakened. We will fix the flow by the IR value of κ. In a realistic model this would correspond to the vev (which can be determined from the Z/W-boson masses)

$$v = \lim_{k \to 0} \sqrt{2\kappa} k.$$

The IR values of the other two parameters are predicted by the RG flow starting from the NGFP in the UV and are related to the Higgs and the Top mass.

$$m_{\rm Higgs} = \sqrt{\lambda_2} v, \quad m_{\rm top} = \sqrt{h^2} v.$$

Choosing the standard model vev $v = 246$GeV the predictions within this toy model are [22]

$$m_{\rm Higgs} = 0.97v \simeq 239\text{GeV}, \quad m_{\rm top} = 5.78v \simeq 1422\text{GeV}.$$

6. Discussion and Conclusions

In this contribution we sketched the idea of asymptotic safety and explained how this scenario could conceptually be applied to solve the problems of triviality and hierarchy in the standard model of particle physics. Therefore we use the functional RG in the formulation by Wetterich as a nonperturbative tool for QFT, and derive flow equations for a chiral Yukawa model with one right-handed and $N_{\rm L}$ left-handed fermions. This asymmetry of the fermion species allows for a balancing of the fermion and the boson fluctuations and therefore can generate a NGFP in theory space. We find NGFPs for $1 \leq N_{\rm L} \leq 57$ and analyse the properties and predictions of asymptotic safety explicitely for the example $N_{\rm L} = 10$. Here we find a NGFP with one relevant and two irrelevant directions in theory space in a basic truncation, allowing for a prediction of the toy Higgs and the toy top mass.

This result is stable with respect to an extension of the truncation in the effective potential. Due to the existence of massless Goldstone and fermion fluctuations, which are not present in the standard model, we observe a possible destabilisation at next-to leading order in the derivative expansion. A more realistic model requires gauge bosons, potentially stabilizing our scenario. Work in this direction is under way. In fact, the NGFP discussed in the present work has first been discovered in a simpler Z_2 invariant Yukawa system, where the discrete symmetry does not give rise to Goldstone bosons in the broken regime [21]. Even though the fixed point exists in the Z_2 model only for somewhat esoteric fermion flavor numbers $N_{\rm f} \lesssim 0.3$, the fixed point and its critical properties have been shown to remain stable also at next-to-leading order in the derivative expansion.

We conclude that an asymptotically safe gauged version of our model has the prospect of quantitatively predicting IR observables such as particle masses, which are free parameters in a perturbative analysis of the standard model. It would solve the problem of triviality structurally and it could improve the hierarchy problem. Put on a gravitational background as in [36] and in connection with the asymptotic safety scenario in quantum gravity this could constitute a fundamental version of all known interactions which is valid on arbitrarily large scales.

7. Acknowledgments

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